INTERNATIONAL BACCALAUREATE
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MARKSCHEME

May 2001

MATHEMATICAL METHODS

Standard Level

Paper 2

17 pages

1. (a) Value $= 1500(1.0525)^3$ ***(M1)***

$= 1748.87$ ***(A1)***

$= 1749$ (nearest franc) ***(A1)***

[3 marks]

(b) $3000 = 1500(1.0525)^t \Rightarrow 2 = 1.0525^t$ ***(M1)***

$t = \dfrac{\log 2}{\log 1.0525} = 13.546$ ***(A1)***

It takes 14 years. ***(A1)***

[3 marks]

(c) $3000 = 1500(1+r)^{10}$ or $2 = (1+r)^{10}$ ***(M1)***

$\Rightarrow \sqrt[10]{2} = 1 + r$ or $\log 2 = 10\log(1+r)$ ***(M1)***

$\Rightarrow \quad r = \sqrt[10]{2} - 1$ or $r = 10^{\frac{\log 2}{10}} - 1$ ***(A1)***

$r = 0.0718$ [or 7.18%] ***(A1)***

[4 marks]

Total [10 marks]

2. (a) $s = 7.41$(3 s.f.) ***(G3)***

[3 marks]

(b)

Weight (W)	$W \le 85$	$W \le 90$	$W \le 95$	$W \le 100$	$W \le 105$	$W \le 110$	$W \le 115$
Number of packets	5	15	**30**	**56**	**69**	**76**	80

(A1)

[1 mark]

(c) (i) From the graph, the median is approximately 96.8.
Answer: 97 (nearest gram). ***(A2)***

(ii) From the graph, the upper or third quartile is approximately 101.2.
Answer: 101 (nearest gram). ***(A2)***

[4 marks]

(d) Sum = 0, since the sum of the deviations from the mean is zero. ***(A2)***

OR

$$\sum (W_i - \overline{W}) = \sum W_i - \left(80 \frac{\sum W_i}{80}\right) = 0$$ ***(M1)(A1)***

[2 marks]

(e) Let A be the event: $W > 100$, and B the event: $85 < W \le 110$

$$\mathrm{P}(A|B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$$ ***(M1)***

$$\mathrm{P}(A \cap B) = \frac{20}{80}$$ ***(A1)***

$$\mathrm{P}(B) = \frac{71}{80}$$ ***(A1)***

$$\mathrm{P}(A|B) = 0.282$$ ***(A1)***

OR

71 packets with weight $85 < W \le 110$. ***(M1)***
Of these, 20 packets have weight $W > 100$. ***(M1)***

Required probability $= \frac{20}{71}$ ***(A1)***

$= 0.282$ ***(A1)***

Note:	Award ***(A2)*** for a correct final answer with no reasoning. Award up to ***(M2)*** for correct reasoning or method.

[4 marks]

Total [14 marks]

3. (a) At $t = 2$, $\begin{pmatrix} 2 \\ 0 \end{pmatrix} + 2\begin{pmatrix} 0.7 \\ 1 \end{pmatrix} = \begin{pmatrix} 3.4 \\ 2 \end{pmatrix}$ ***(M1)***

Distance from $(0, 0) = \sqrt{3.4^2 + 2^2} = 3.94$ m ***(A1)***

[2 marks]

(b) $\left|\begin{pmatrix} 0.7 \\ 1 \end{pmatrix}\right| = \sqrt{0.7^2 + 1^2}$ ***(M1)***

$= 1.22 \text{ ms}^{-1}$ ***(A1)***

[2 marks]

(c) $x = 2 + 0.7t$ and $y = t$ ***(M1)***

$x - 0.7y = 2$ ***(A1)***

[2 marks]

(d) $y = 0.6x + 2$ and $x - 0.7y = 2$ ***(M1)***

$x = 5.86$ and $y = 5.52$ $\left(\text{or } x = \frac{170}{29} \text{ and } y = \frac{160}{29}\right)$ ***(A1)(A1)***

[3 marks]

(e) The time of the collision may be found by solving

$\begin{pmatrix} 5.86 \\ 5.52 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0.7 \\ 1 \end{pmatrix} t$ for t ***(M1)***

$\Rightarrow t = 5.52$ s ***(A1)***

[*i.e.* collision occurred 5.52 seconds after the vehicles set out].

Distance d travelled by the motorcycle is given by

$d = \left|\begin{pmatrix} 5.86 \\ 5.52 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \end{pmatrix}\right| = \sqrt{(5.86)^2 + (3.52)^2}$ ***(M1)***

$= \sqrt{46.73}$

$= 6.84$ m ***(A1)***

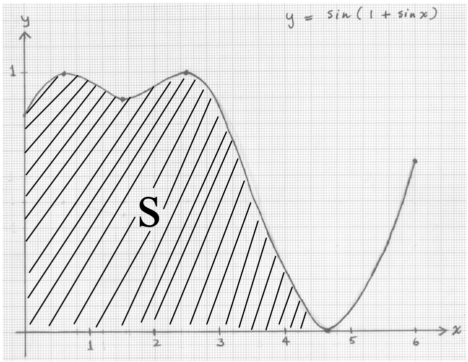
Speed of the motorcycle $= \frac{d}{t} = \frac{6.84}{5.52}$

$= 1.24 \text{ ms}^{-1}$ ***(A1)***

[5 marks]

Total [14 marks]

4. (a) (i) *(A4)*



Notes:	Only a rough sketch of the graph is required (no scales necessary). Award ***(A1)*** for any one (local) maximum. Award ***(A1)*** for the minimum at $\frac{\pi}{2}$, ***(A1)*** for the second minimum.

(ii) Maximum/minimum points at: 0.6075, 1.571, 2.534, 4.712 ***(G1)(G1)(G1)(G1)(A1)***

Note:	Award the ***(A1)*** if **all four** answers are correct to 4 s.f.

[9 marks]

(b) (i) See graph ***(A1)***

(ii) $\int_0^{\frac{3\pi}{2}} \sin(1+\sin x)\mathrm{d}x$ or $\int_0^{4.712} \sin(1+\sin x)\mathrm{d}x$ ***(A2)***

(iii) 3.517 ***(G2)***

[5 marks]

(c) For all x, $-1 \le \sin x \le 1$; hence $0 \le 1+\sin x \le 2$. ***(R1)***

On the interval $[0, 2]$ $\sin x \ge 0$; hence $\sin(1+\sin x) \ge 0$ ***(R1)***

[2 marks]

Total [16 marks]

5. (a) (i) $\text{AP} = \sqrt{(x-8)^2 + (10-6)^2} = \sqrt{x^2 - 16x + 80}$ ***(M1)(AG)***

(ii) $\text{OP} = \sqrt{(x-0)^2 + (10-0)^2} = \sqrt{x^2 + 100}$ ***(A1)***

[2 marks]

(b) $\cos \text{O}\hat{\text{P}}\text{A} = \dfrac{\text{AP}^2 + \text{OP}^2 - \text{OA}^2}{2\text{AP} \times \text{OP}}$ ***(M1)***

$= \dfrac{(x^2 - 16x + 80) + (x^2 + 100) - (8^2 + 6^2)}{2\sqrt{x^2 - 16x + 80}\sqrt{x^2 + 100}}$ ***(M1)***

$= \dfrac{2x^2 - 16x + 80}{2\sqrt{x^2 - 16x + 80}\sqrt{x^2 + 100}}$ ***(M1)***

$\cos \text{O}\hat{\text{P}}\text{A} = \dfrac{x^2 - 8x + 40}{\sqrt{\left\{(x^2 - 16x + 80)(x^2 + 100)\right\}}}$ ***(AG)***

[3 marks]

(c) For $x = 8, \cos \text{O}\hat{\text{P}}\text{A} = 0.780869$ ***(M1)***

$\arccos 0.780869 = 38.7^\circ$ (3 s.f.) ***(A1)***

OR

$\tan \text{O}\hat{\text{P}}\text{A} = \dfrac{8}{10}$ ***(M1)***

$\text{O}\hat{\text{P}}\text{A} = \arctan(0.8) = 38.7^\circ$ (3 s.f.) ***(A1)***

[2 marks]

(d) $\text{O}\hat{\text{P}}\text{A} = 60^\circ \Rightarrow \cos \text{O}\hat{\text{P}}\text{A} = 0.5$

$0.5 = \dfrac{x^2 - 8x + 40}{\sqrt{\left\{(x^2 - 16x + 80)(x^2 + 100)\right\}}}$ ***(M1)***

$2x^2 - 16x + 80 - \sqrt{\left\{(x^2 - 16x + 80)(x^2 + 100)\right\}} = 0$ ***(M1)***

$x = 5.63$ ***(G2)***

[4 marks]

(e) (i) $f(x) = 1$ when $\cos \text{O}\hat{\text{P}}\text{A} = 1$ ***(R1)***

hence, when $\text{O}\hat{\text{P}}\text{A} = 0$. ***(R1)***

This occurs when the points O, A, P are collinear. ***(R1)***

(ii) The line (OA) has equation $y = \dfrac{3x}{4}$ ***(M1)***

When $y = 10, x = \dfrac{40}{3}\left(= 13\tfrac{1}{3}\right)$ ***(A1)***

OR

$x = \dfrac{40}{3}\left(= 13\tfrac{1}{3}\right)$ ***(G2)***

Note: Award ***(G1)*** for 13.3.

[5 marks]

Total [16 marks]

6. (i)

Diagram	Correlation coefficient
(a)	0.50
(b)	–0.60
(c)	0.90
(d)	–0.95
(e)	0.00

(A4)

> **Note:** Award ***(A1)*** for each correct answer, if not all answers correct.

[4 marks]

(ii) (a) Regression line (after scaling down x and y by a factor of 1000)
$y = 24250 + 18.5x$ (or $y = 24.25 + 18.5x$) ***(G3)***
for $x = 7000$, $y = 153750$ (or for $x = 7$, $y = 153.75$) ***(A1)***
Predicted sales: 154,000 dollars (to the nearest 1000 dollars) ***(A1)***
[5 marks]

(b) A change of scale $x' = kx$ and $y' = ky$ affects the mean and the standard deviation but not the correlation coefficient nor the gradient of the regression line.
$\bar{x}$ changes ***(A1)***
s_x changes ***(A1)***
r remains the same ***(A1)***
the gradient of the regression line remains the same ***(A1)***
[4 marks]

(iii) (a) Let X be the random variable for the IQ.
$X \sim \text{N}(100, 225)$
$\text{P}(90 < X < 125) = \text{P}(-0.67 < Z < 1.67)$ ***(M1)***
$= 0.701$
70.1 percent of the population (accept 70 percent). ***(A1)***

OR

$\text{P}(90 < X < 125) = 70.1\%$ ***(G2)***
[2 marks]

(b) $\text{P}(X \geq 125) = 0.0475$ (or 0.0478) ***(M1)***
P(both persons having IQ ≥ 125) $= (0.0475)^2$ $\left(\text{or } (0.0478)^2\right)$ ***(M1)***
$= 0.00226$ (or 0.00228) ***(A1)***
[3 marks]

(c) Null hypothesis (H_0): mean IQ of people with disorder is 100 ***(M1)***
Alternative hypothesis (H_1): mean IQ of people with disorder is less than 100 ***(M1)***

$$\text{P}(\bar{X} < 95.2) = \text{P}\left(Z < \left(\frac{95.2 - 100}{\frac{15}{\sqrt{25}}}\right)\right) = \text{P}(Z < -1.6) = 1 - 0.9452$$
$$= 0.0548$$ ***(A1)***

The probability that the sample mean is 95.2 and the null hypothesis true is 0.0548 > 0.05. Hence the evidence is not sufficient. ***(R1)***
[4 marks]

continued…

Question 6 continued

(iv) (a)

	Accept offer	Reject offer
Students	35.2	52.8
Teachers	20.8	31.2

(M1)(A2)

Note: Award ***(M1)*** for row and column totals, ***(A2)*** for correct calculations.

[3 marks]

(b) $\chi^2 = 4.90$ ***(G2)***

[2 marks]

(c) For 1 degree of freedom, the critical value is 6.64 ***(A1)***
$4.90 < 6.64$ ***(R1)***
Conclusion (iii) ***(A1)***

OR

$\mathrm{P}(4.90) = 0.0269$ ***(G1)***
$0.0269 > 0.01$ ***(R1)***
Conclusion (iii) ***(A1)***

[3 marks]

Total [30 marks]

7. (a) (i)
$$f'(x) = \frac{\left(x \times \frac{1}{2x} \times 2\right) - (\ln 2x \times 1)}{x^2}$$ ***(M1)(M1)***

Note: Award ***(M1)*** for the correct use of the quotient rule and ***(M1)*** for correct substitution.

$$= \frac{1 - \ln 2x}{x^2}$$ ***(AG)***

(ii) $f'(x) = 0$ for max/min. ***(R1)***

$\frac{1 - \ln 2x}{x^2} = 0$ only at 1 point, when $x = \frac{e}{2}$ ***(R1)***

Note: Award no marks if the reason given is of the sort "by looking at the graph".

(iii) Maximum point when $f'(x) = 0$.

$f'(x) = 0$ for $x = \frac{e}{2}$ $(= 1.36)$ ***(A1)***

$y = f\left(\frac{e}{2}\right) = \frac{2}{e}$ $(= 0.736)$ ***(A1)***

Note: Award ***(A1)*** per correct coordinate if the answer is found using the GDC, regardless of method. If one or both coordinates are wrong, you may award up to 1 mark for method.

[6 marks]

(b)
$$f''(x) = \frac{-\frac{1}{2x} \times 2 \times x^2 - (1 - \ln 2x)2x}{x^4}$$ ***(M1)(M1)***

$$= \frac{2\ln 2x - 3}{x^3}$$ ***(AG)***

Inflexion point $\Rightarrow f''(x) = 0$ ***(M1)***

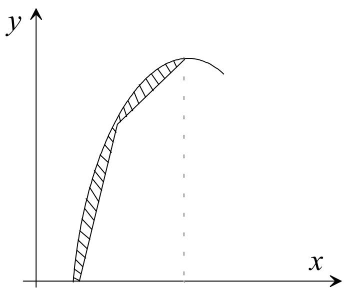
$\Rightarrow 2\ln 2x = 3$ ***(M1)***

$x = \frac{e^{1.5}}{2}$ $(= 2.24)$ ***(A1)***

$y = f\left(\frac{e^{1.5}}{2}\right) = \frac{3}{e^{1.5}}$ $(= 0.669)$ ***(A1)***

[6 marks]

(c) (i) The trapezium rule would underestimate the area of *S*. ***(A1)***



Shaded area not included when using the trapezium rule (or similar reasonable explanation). ***(R2)***

[3 marks]

continued…

Question 7 (c) continued

(ii) $u = \ln 2x;\ \mathrm{d}u = \frac{1}{2x} \times 2\mathrm{d}x = \frac{1}{x}\mathrm{d}x$ ***(M1)***

$\int \frac{\ln 2x}{x}\mathrm{d}x = \int u\,\mathrm{d}u$ ***(M1)***

$= \frac{u^2}{2} + C$ ***(A1)***

$= \frac{(\ln 2x)^2}{2} + C$ ***(A1)***

[4 marks]

(iii) Area of $S = \int_{0.5}^{\frac{e}{2}} \frac{\ln 2x}{x}\mathrm{d}x$ ***(M1)(A1)***

Note: Award ***(M1)*** for the integral expression, and ***(A1)*** for the limits.

(M1)

$= \frac{\left(\ln 2\left(\frac{e}{2}\right)\right)^2}{2} - \frac{\left(\ln(2 \times 0.5)\right)^2}{2}$ ***(M1)***

$= \frac{1}{2} - 0 = \frac{1}{2}$ ***(A1)***

Note: Award only ***(A1)(M0)(M0)(A1)*** if the area (to 3 s.f. or exactly) is found on the GDC.

[4 marks]

(d) (i) If $x_1 = 1$, then $x_2 = -1.26$ ***(M1)***

$f(x_2) = f(-1.26)$ does not exist, so x_3 cannot be calculated. ***(R2)***

[3 marks]

(ii) $x_2 = 0.4 - \frac{f(0.4)}{f'(0.4)} = 0.47297$ ***(A1)***

Absolute error $= |0.5 - 0.47297| = 0.02703$ ***(A1)***

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.49787$ ***(A1)***

Absolute error $= |0.5 - 0.49787| = 0.00213$ ***(A1)***

which is less than 0.01.

Note: Absolute errors need not be explicitly given. Award ***(A3)*** if further terms are listed, without stating that they are unnecessary.

[4 marks]

Total [30 marks]

8. (a) (i) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} -12 & -15 \\ -6 & -3 \end{pmatrix}$ ***(M1)***

$\Rightarrow 2a = -12;\ a + 3b = -15;\ 2c = -6;\ c + 3d = -3$ ***(M1)***

$\boldsymbol{T} = \begin{pmatrix} -6 & -3 \\ -3 & 0 \end{pmatrix}$ ***(C2)***

OR

$\boldsymbol{T} = \begin{pmatrix} -12 & -15 \\ -6 & -3 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}^{-1}$ ***(M1)***

$= \begin{pmatrix} -12 & -15 \\ -6 & -3 \end{pmatrix}\begin{pmatrix} \frac{1}{2} & -\frac{1}{6} \\ 0 & \frac{1}{3} \end{pmatrix}$ ***(M1)***

$\boldsymbol{T} = \begin{pmatrix} -6 & -3 \\ -3 & 0 \end{pmatrix}$ ***(C2)***

Note: Award ***(M1)*** for any correct method. Award ***(C3)*** for the correct answer.

[4 marks]

(ii) C′ is the image of C under ***T***. Hence, C′ has coordinates (3, 3). ***(A1)***

[1 mark]

(b) (i) $\boldsymbol{V}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 - 6x - 3y \\ -1 - 3x \end{pmatrix}$ ***(M1)***

$x = 1 - 6x - 3y$

$y = -1 - 3x$ ***(M1)***

$x = -2;\ y = 5$ ***(A1)(A1)***

OR

$x = -2\ y = 5$ ***(G3)***

[4 marks]

(ii) Areas under ***V*** are multiplied by $\left|\det\begin{pmatrix} -6 & -3 \\ -3 & 0 \end{pmatrix}\right| = 9$ ***(M1)***

Note: Translation by the vector (1, −1) does not change areas.

Hence, the image of *D* has area 9*a*. ***(A1)***

Note: Award ***(A2)*** for the correct answer, irrespective of method.

[2 marks]

(iii) $\boldsymbol{L} = \boldsymbol{E}\ \boldsymbol{S}\ \boldsymbol{F}$

$\begin{pmatrix} -6 & -3 \\ -3 & 0 \end{pmatrix} = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}\begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ ***(M1)***

$\boldsymbol{E} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix};\ \boldsymbol{S} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix};\ \boldsymbol{F} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ ***(A1)(A1)(A1)***

Notes: Award ***(A1)*** for each correct matrix irrespective of method. Award up to ***(M1)*** for method.

[4 marks]

continued…

Question 8 continued

(c) (i) The image of (x, y) under $\boldsymbol{W}$ is $(0, y)$. ***(A1)***

[1 mark]

(ii) The image of the triangle ABC under $\boldsymbol{W}$ is the line segment joining (0, 0) to (0, 3). ***(A2)***

Note: If answer incorrect, award up to ***(M1)*** for method.

[2 marks]

(iii) $\boldsymbol{QR}$ is the rotation through $60^\circ - 45^\circ = 15^\circ$ counterclockwise. ***(M1)(M1)***
Hence, its matrix is

$$\begin{pmatrix} \cos 15^\circ & -\sin 15^\circ \\ \sin 15^\circ & \cos 15^\circ \end{pmatrix}$$ ***(M2)***

$$= \begin{pmatrix} 0.966 & -0.259 \\ 0.259 & 0.966 \end{pmatrix}$$ ***(AG)***

OR

$$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{\left(\sqrt{2}+\sqrt{6}\right)}{4} & \frac{\left(\sqrt{2}-\sqrt{6}\right)}{4} \\ \frac{\left(\sqrt{6}-\sqrt{2}\right)}{4} & \frac{\left(\sqrt{2}+\sqrt{6}\right)}{4} \end{pmatrix}$$ ***(M1)(M1)(M2)***

$$= \begin{pmatrix} 0.966 & -0.259 \\ 0.259 & 0.966 \end{pmatrix}$$ ***(AG)***

[4 marks]

(iv) $\det \boldsymbol{U} = \det \boldsymbol{WQR} = (\det \boldsymbol{W})(\det \boldsymbol{QR}) = 0$ since $\det(\boldsymbol{W}) = 0$. ***(M1)***
Since $\det \boldsymbol{U} = 0$, $\boldsymbol{U}$ cannot be an isometry. ***(R2)***

OR

Since $\boldsymbol{QR}$ is an isometry (rotation about the origin) and $\boldsymbol{W}$ is not, their composition $\boldsymbol{WQR} = \boldsymbol{U}$ is not an isometry. ***(R3)***

OR

The distance between two points is not preserved. ***(R3)***

[3 marks]

(v) $$\boldsymbol{U}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0.259x + 0.966y \end{pmatrix}$$ ***(M1)***

$$\boldsymbol{QR}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.966x - 0.259y \\ 0.259x + 0.966y \end{pmatrix}$$ ***(M1)***

hence, $\boldsymbol{U}(x, y) = \boldsymbol{QR}(x, y) \Leftrightarrow 0 = 0.966x - 0.259y$ ***(M2)***

Answer: the line $y = 3.73x$, or $y = \tan 75^\circ x$, or $0.966x - 0.259y = 0$ ***(A1)***

[5 marks]

Total [30 marks]